University of Pavia IUSS School of Advanced Studies

Civil Engineering for Mitigation of Risk from Natural Hazards

Course: Applied Mathematics
A.Y.: 2021/2022
Dates of classes: 18/11/2021 - 17/12/2021
Final exam: 21-22/12/2021
Lecturer: Lorenzo Tamellini
Teaching Assistant: Daniele Prada
Classroom: EUCentre/online

Week	Date	Lecture hours	Tutorial hours	Subject	Tot h
1	18/11/2020	09:30/12:30	13:30/15:30	**CLASS:** unconstrained optimization **EXER:** Matlab basics, if/for/while statements, basic algorithms (bubblesort, bisection, Tomas)	5
	19/11/2020	---	---	---	---
2	22/11/2020	---	---	---	---
	23/11/2020	09:30/12:30	13:30/15:30	**CLASS:** constrained opt, numerical methods for optimization (Finite Differences, Gradient) **EXER:** functions, @-functions**,** basic algorithms (bubblesort, bisection, Tomas)	5
	24/11/2020	13:30/15:30	09:30/12:30	**CLASS:** introduction to ODE, linear dynamical systems **EXER:** optimization **HW1 (Due date: Nov. 30)**	5
	25/11/2020	**09:30/13:30**		**CLASS:** stability linear/nonlinear dynamical systems, bifurcations	4
	26/11/2020	---	---	---	---
3	29/11/2020	---	---	---	---
	30/11/2020	13:30/15:30	09:30/12:30	**EXER:** ODE **HW2 (Due date: Dec. 7)** **CLASS:** L^2 spaces, orthogonal polynomials, Legendre polynomials	5
	01/12/2020	09:30/12:30	13:30/15:30	**CLASS:**interpolation, least-squares Fourier expansion (real form) **EXER:** orthogonal pol, interpolation, least squares approx.	5
	02/12/2020	09:30/11:30	**11:30/13:30** **14:30/16:30**	**CLASS:** Fourier expansion (complex form), DFT **EXER:** Fourier expansion, Matlab's FFT,	6
	03/12/2020	---	---	---	---
4	06/12/2020	---	---	---	---
	07/12/2020	09:30/12:30	13:30/15:30	**CLASS:** Fourier transform; Dirac's delta **EXER:** Fourier transform **HW3 (Due date: Dec. 14)**	5
	08/12/2020	---	---	*(holiday)*	---
	09/12/2020	---	---	*(holiday)*	---
	10/12/2020	---	---	---	---
5	13/12/2020	---	---	---	---
	14/12/2020	09:30/12:30	13:30/15:30	**CLASS:** intro to PDE, finite differences for elliptic PDEs **EXER**: finite differences **HW4 (Due date Date: Dec. 17)**	5
	15/12/2020	09:30/12:30	13:30/15:30	**CLASS:** elliptic PDE: mean,smoothing, heat PDE: fundamental solution, Duhamel principle. separation of variables **EXER:** separation of variables	5
	16/12/2020	09:30/12:30	13:30/15:30	**CLASS:** linear transport PDE, wave PDE **EXER:** linear transport PDE, wave PDE	5
	17/12/2020	---	---	---	---

OBJECTIVES: To provide advanced mathematical tools that will be used throughout the rest of the program.

DESCRIPTION:

The course is divided into five chapters as follows. Each chapter also includes exercises and hands-on Matlab sessions (32 hours of lectures + 23h hours of exercises/Matlab sessions).

1. **Introduction to Matlab and basic programming (4h exercises/Matlab).** Basic Matlab commands, if/for/while instructions. Functions and @-functions. Design of basic algorithms.
2. **Optimization of N-variate functions (6h lectures + 2h exercises/Matlab = 8h).** Free and constrained optimization of N-variate functions. Lagrange multipliers and KKT conditions. Optimization algorithms (Gradient, Newton, finite differences)
3. **Ordinary Differential Equations (ODE) (7h lectures + 3h exercises/Matlab = 10h).** Scalar ODEs and system of ODEs. Analytic solutions of linear systems of ODEs (exponential matrix). Equilibria of linear and non-linear systems (linearization, Lyapunov's function) and bifurcations.
4. **Function approximation and Fourier transform (10h lectures + 8h exercises/Matlab = 18h).** Space of square-summable functions, orthonormal bases and Parseval's identity, Fourier and Legendre expansions, interpolation and least squares approximation. Fourier transform, DFT/FFT, Dirac's delta.
5. **Partial Differential Equations (PDE) (9h lectures + 6 exercises/Matlab = 15h).**
    a. **Elliptic and parabolic PDEs:** separation of variables, maximum and mean principle, smoothing property. Fundamental solution of heat equation. Separation of variables, finite differences.
    b. **Hyperbolic PDEs:** method of lines for 1st order hyperbolic PDEs, inflow and outflow; D'Alambert formula for wave equation on the line and semiline, separation of variables

REFERENCES:

Class notes made available during the course. For backup and further readings:

- **Optimization of N-variate functions (Ch. 2):** J. Nocedal, S. Wright. Numerical Optimization. Springer;
- **Ordinary Differential Equations (Ch. 3):** G. Teschl, Ordinary Differential Equations and Dynamical Systems, American Mathematical Society; Blanchard, Devaney, Hall. Differential Equations, Cengage Learning.
- **Function approximation, transforms (Ch. 4), programming of numerical algorithms (Ch. 1):** A. Quarteroni, R. Sacco, F. Saleri. Numerical Mathematics. Springer; D. Kammler, A First Course in Fourier Analysis, Cambridge University Press;
- **Partial Differential Equations (Ch. 5):** S. Salsa, Partial Differential Equations in Action, Springer; L. Evans, Partial Differential Equations. American Mathematical Society

Italian-speaking students can also use these books:

- **Chapters 2,3,4:** Analisi Matematica 2, M. Bramanti, C. Pagani, S. Salsa, Zanichelli ed.;
- **Chapter 5:** Equazioni a Derivate Parziali – Metodi, modelli e applicazioni, S. Salsa, Springer;

ASSESSMENT:

The final grade will be composed as follows:

- 30% homework assignments (four in total) graded during the course;
- 70% oral discussion over the content of the class (theory, exercises, matlab scripts)

COURSE WEBSITE: https://sites.google.com/view/tamellini-applied-mathematics