Course: Applied Mathematics
A.Y.: 2022/2023
Date of classes: 28/11/2022 – 21/12/2022
Final exam: 23/12/2023
Lecturer: Massimiliano Martinelli (martinelli@imati.cnr.it)
Classroom: Eucentre room 1
Last update: 28/11/2022

Week	Date	Lecture hours	Subject	Tot h
1	28/11/22	10:00/13:00	Topics on linear algebra, complex numbers and real analysis	3
	29/11/22	10:00/13:00	Eigenvalues and eigenvectors. Matrix diagonalization. Introduction to Matlab	3
	30/11/22	10:00/13:00	Unconstrained optimization	3
	01/12/22	10:00/13:00	Constrained optimization, numerical methods for optimization	3
	02/12/22	14:00/17:00	Introduction to ODEs.	3
2	05/12/22	10:00/13:00	Linear dynamical systems	3
	06/12/22	10:00/13:00	Study of the harmonic oscillator (damped and with external force)	3
	07/12/22	10:00/13:00	Stability of linear/nonlinear dynamical systems	3
	08/12/22	---	(holiday)	
	09/12/22	---	(holiday)	
3	12/12/22	10:00/13:00	L^2 spaces, orthogonal polynomials, Legendre polynomials	3
	13/12/22	10:00/13:00	Interpolation, least-squares, Fourier expansion (real form)	3
	14/12/22	10:00/13:00	Fourier expansion (complex form), DFT	3
	15/12/22	10:00/13:00	Fourier transform; Dirac's delta	3
	16/12/22	14:00/17:00	Introduction to PDEs. Elliptic PDEs and finite differences	3
4	19/12/22	---	---	
	20/12/22	10:00/13:00	Heat PDE. Separation of variables.	3
	21/12/22	10:00/13:00	Linear transport PDE, wave PDE	3
	22/12/22	---	---	
	09/01/23	TBD	Final Exam (Written test)	
				45

OBJECTIVES: To provide advanced mathematical tools that will be used throughout the rest of the program.

DESCRIPTION: The course is divided into four chapters as follows.

1) **Optimization of N-variate functions.** Free and constrained optimization of N-variate functions. Lagrange multipliers and KKT conditions. Optimization algorithms (Gradient, Newton, finite differences)
2) **Ordinary Differential Equations (ODE)** Scalar ODEs and system of ODEs. Analytic solutions of linear systems of ODEs (exponential matrix). Study of the harmonic oscillator (damped and with external force). Equilibria of linear and non-linear systems (linearization, Lyapunov's function).
3) **Function approximation and Fourier.** Space of square-summable functions, orthonormal bases and Parseval's identity, Fourier and Legendre expansions, interpolation and least squares approximation. Fourier transform, DFT/FFT, Dirac's delta.
4) **Partial Differential Equations (PDE)**
   a) **Elliptic and parabolic PDEs:** separation of variables, maximum and mean principle, smoothing property. Fundamental solution of heat equation. Separation of variables, finite differences.
   b) **Hyperbolic PDEs: method of lines for 1st order hyperbolic PDEs, inflow and outflow;** D'Alambert formula for wave equation on the line and semiline, separation of variables**.**

MATLAB will be used during the classes to provide examples of the discussed topics.

REFERENCES: Class notes made available during the course. For backup and further readings:

- **Optimization of N-variate functions (Ch. 1):** J. Nocedal, S.Wright. Numerical Optimization. Springer;
- **Ordinary Differential Equations (Ch. 2):** G. Teschl, Ordinary Differential Equations and Dynamical Systems, American Mathematical Society; Blanchard, Devaney, Hall. Differential Equations, Cengage Learning.
- **Function approximation, transforms (Ch. 3):** A. Quarteroni, R. Sacco, F. Saleri. Numerical Mathematics. Springer; D. Kammler, A First Course in Fourier Analysis, Cambridge University Press;
- **Partial Differential Equations (Ch. 4):** S. Salsa, Partial Differential Equations in Action, Springer; L. Evans, Partial Differential Equations. American Mathematical Society
- **MATLAB:** MATLAB Primer (https://it.mathworks.com/help/pdf_doc/matlab/learn_matlab.pdf), MATLAB Programming Fundamentals (https://www.mathworks.com/help/pdf_doc/matlab/matlab_prog.pdf),

Italian-speaking students can also use these books:

- **Chapters 1, 2, 3:** Analisi Matematica 2, M. Bramanti, C. Pagani, S. Salsa, Zanichelli ed.;
- **Chapter 4:** Equazioni a Derivate Parziali – Metodi, modelli e applicazioni, S. Salsa, Springer;

ASSESSMENT: The final grade will be given after a written exam over the content of the class (theory, exercises).

COURSE WEBSITE: https://elearning.unipv.it/course/view.php?id=3485